MJC-3 Date :- 06/08/2025

B. Sc, Sem III

~~Sequence~~

Sequences

A sequence of real numbers is defined as a function $S : \mathbb{N} \longrightarrow \mathbb{R}$, then for each $n \in \mathbb{N}$, $S(n)$ or S_n is a real number. The real numbers $S_1, S_2, S_3, \ldots\ldots, S_n$ are called terms of sequence. A sequence may be written as $\{S_1, S_2, S_3, \ldots, S_n\}$ or $\{S_n\}$.

For example

$\mathbb{N} = \{1, 2, 3, 4, \ldots, \ldots\}$

$\mathbb{R}$ = set of real numbers

1. $\{S_n\} = \{(-1)^n\}, n \in \mathbb{N}$

Here $S_1 = (-1)^1 = -1$, $S_2 = (-1)^2 = 1$, $S_3 = (-1)^3 = -1$,

$S_4 = (-1)^4 = 1, \ldots$

Hence $$S_n = \begin{cases} -1 & \text{if } n \text{ is odd} \\ 1 & \text{if } n \text{ is even} \end{cases}$$

2. $\{S_n\} = \{\frac{1}{n}\}, n \in \mathbb{N}$

Here, $S_1 = \frac{1}{1} = 1$, $S_2 = \frac{1}{2}$, $S_3 = \frac{1}{3}$, $S_4 = \frac{1}{4}, \ldots, S_{100} = \frac{1}{100}, \ldots$

3. $\{S_n\} = \{(1 + \frac{1}{n})^n\}, n \in \mathbb{N}$

$S_1 = (1+\frac{1}{1})^1 = 2$, $S_2 = (1+\frac{1}{2})^2 = \frac{9}{4}$, $S_3 = (1+\frac{1}{3})^3 = \cancel{\frac{16}{27}}$ $= \frac{64}{27}$,

$S_4 = (1+\frac{1}{4})^4 = \frac{5^4}{4^4} = \frac{625}{256}, \ldots\ldots$ etc

4) $\{S_n\}$, where $S_n = 1 + (-1)^n,\ n \in \mathbb{N}$

Here $S_1 = 1+(-1)^1 = 1+(-1) = 0,\ S_2 = 1+(-1)^2 = 1+1 = 2,$

$S_3 = 1+(-1)^3 = 1+(-1) = 0,\ S_4 = 1+(-1)^4 = 2, \dots$

Hence

$$S_n = \begin{cases} 0 & \text{if } n = \text{odd} \\ 2 & \text{if } n = \text{even} \end{cases}$$

5. $\{S_n\}$, where $S_n = 1,\ \forall\, n \in \mathbb{N}$

$S_1 = 1,\ S_2 = 1,\ S_3 = 1,\ S_4 = 1, \dots$

6. $\{S_n\} = \left\{ \frac{(-1)^{n-1}}{n!} \right\},\ n \in \mathbb{N}$

Here $S_1 = \frac{(-1)^{1-1}}{1!} = \frac{(-1)^0}{1} = 1,\ S_2 = \frac{(-1)^{2-1}}{2!} = \frac{(-1)^1}{2} = \frac{-1}{2}$

$S_3 = \frac{(-1)^2}{3!} = \frac{1}{1\times2\times3} = \frac{1}{6},\ S_4 = \frac{(-1)^3}{4!} = \frac{-1}{24}, \dots$

Home work

Write first five terms of the following sequence

(1) $\{S_n\} = \{-5n\},\ n \in \mathbb{N}$ (2) $\{S_n\} = \left\{\frac{n}{n+1}\right\}$,

(3) $\{S_n\} = \sqrt{3S_n}$ (4) $\{S_n\} = \left\{ \sin\left(\frac{n\pi}{2}\right) + \frac{(-1)^n}{n} \right\},\ n \in \mathbb{N}$

Convergence of sequences

A sequence $\{S_n\}$ is said to converge to a real number l if $\forall\, \varepsilon > 0,\ \exists\, n_0 \in \mathbb{N}$, ($n_0$ depending on ε) such that

$|S_n - l| < \varepsilon,\ \forall\, n \geq n_0$.

Symbolically, $S_n \to l$ as $n \to \infty$ or $\lim_{n\to\infty} S_n = l$.